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OPTIMAL DAMPING BETWEEN TWO ADJACENT ELASTIC STRUCTURES

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SUMMARY

The optimal values for the distribution of passive dampers interconnecting two adjacent structures of different heights are determined. The dampers are selected to minimize the seismic response in the first and second modes of the taller of the two structures. For simplicity, the structures are represented as uniform damped shear beams subjected to a common ground motion. Under certain conditions, apparent damping ratios as high as 12 and 15 per cent can be achieved in the first and second modes of lightly damped structures by the introduction of interconnection dampers. The largest reduction of the response in the first mode is achieved when the taller structure is about twice the height of the second structure.

KEY WORDS: seismic response; damping; optimal; structures; dynamics; control

INTRODUCTION

The possibility has been raised of using active or passive control devices interconnecting adjacent structures to reduce the seismic or wind response of one or both structures. This approach can also be used to reduce the problem of pounding between adjacent structures. Since 1972, Klein and co-workers[1–3] have studied the use of dissipative links and semi-active devices to control the response of adjacent buildings to wind excitation. In particular, Gurley *et al.*[3] have considered a system of two adjacent buildings, modelled by uniform shear beams, coupled by a single flexible and damped link. The optimal stiffness and damping of the link when the primary structure was subjected to wind excitation was obtained. They found that damping in the link acting alone tends to increase the response of the primary structure and that link damping used in combination with link stiffness hinders the response reduction obtained by coupling both structures. Consequently, they concluded that the connection should be designed for minimum damping.

The possibility of controlling the seismic response of two or more structures, each modelled as an undamped single-degree-of-freedom system, by a viscous damper or by an active device has been explored by Iwanami *et al.*,[4] Seto *et al.*[5] and Mitsuta *et al.*[6] In particular, Iwanami *et al.*[4] found the optimal link damping for the particular case $m_1k_1 = m_2k_2$ in which the peak amplitudes of the transfer functions for both structures are equal.

Recently, Luco and Wong[7] have obtained the optimal control forces interconnecting two structures each modelled as a discrete multi-degree-of-freedom damped shear beam. In the case of uniform (but dissimilar) structures, the control forces obtained by application of the instantaneous optimal control approach of Yang *et al.*[8] were shown to be proportional to the relative velocity between the attachment points in both structures. Thus, the optimal control forces could be implemented in the form of passive viscous dampers. In

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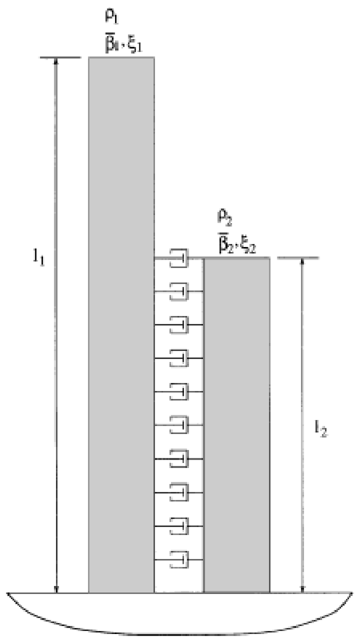

Figure 1. Model of two uniform structures interconnected by uniformly distributed viscous dampers

addition, Luco and Wong[7] found that the damping constants would be uniform if the structures were uniform. In this paper we determine the actual optimal values for the interconnecting damping constants as a function of the properties of both structures and, in particular, as a function of their relative heights. We also determine the optimum height for the second (shorter) structure to minimize the response of the first (taller) structure.

The structural model considered here is illustrated in Figure 1 and consists of two uniform, elastic, continuous and damped shear beams of heights l_1 and l_2 ($l_1 \geqslant l_2$), respectively. The two structures are interconnected by viscous dampers uniformly distributed over the height l_2 of the lower structure and are subjected to a common horizontal ground motion u_g. The effects of soil–structure interaction and the possible lateral variation of the ground motion are ignored in this first study. The response of both structures to harmonic motion of the base is obtained first and the optimal values for the interconnecting damping constants are determined by minimizing the peak amplitude of the transfer function for the response at the top of the taller structure in the vicinity of the first and second modes of vibration. The optimal interconnecting damping constants are then determined for a class of adjacent structures with different relative heights and relative floor masses. Results in the frequency and time domains illustrating the beneficial effects of interconnecting damping are then presented and discussed.

BASIC EQUATIONS

We consider first the harmonic vibrations of two uniform shear beams with time dependence $e^{i\omega t}$ where ω is the circular frequency. The equations of motion in terms of the absolute horizontal displacements $u_1(z)e^{i\omega t}$ and $u_2(z)e^{i\omega t}$ can be written in the form

$$(\beta_1/\bar{\beta}_1)^2 u_1'' + \bar{\omega}^2 u_1 = 0, \qquad 1 < \bar{z} < \bar{l}_1 \tag{1a}$$

$$(\beta_1/\bar{\beta}_1)^2 u_1'' - i\bar{\omega}\pi\xi_o(u_1 - u_2) + \bar{\omega}^2 u_1 = 0, \qquad 0 < \bar{z} < 1 \tag{1b}$$

$$(\beta_2/\bar{\beta}_1)^2 u_2'' - i\bar{\omega}\pi\xi_o(\rho_1/\rho_2)(u_2 - u_1) + \bar{\omega}^2 u_2 = 0, \qquad 0 < \bar{z} < 1 \tag{2}$$

where the prime denotes derivative with respect to $\bar{z}=z/l_2$, $\xi_o = cl_2/\pi\rho_1\bar{\beta}_1$ is a dimensionless interconnecting damping constant, c is the damping constant per unit length of the connecting distributed viscous dampers, ρ_j is the mass per unit length of the jth beam ($j=1,2$), $\bar{\omega}=\omega l_2/\bar{\beta}_1$ is a dimensionless frequency, $\bar{l}_1 = l_1/l_2$ and β_j is the complex shear wave velocity

$$\beta_j = \bar{\beta}_j\,[1+2\,\mathrm{i}\,\xi_j\,\mathrm{sgn}\,(\omega)]^{1/2} \tag{3}$$

in which $\bar{\beta}_j$ is (approximately) the real wave velocity in the jth beam and ξ_j is the hysteretic (structural) damping ratio.

The general solution to equations (1a), (1b) and (2) is

$$u_1(z) = E\cos[\bar{\omega}(\bar{l}_1-\bar{z})\bar{\beta}_1/\beta_1] + F\sin[\bar{\omega}(\bar{l}_1-\bar{z})\bar{\beta}_1/\beta_1], \qquad 1\leqslant\bar{z}\leqslant\bar{l}_1 \tag{4a}$$

$$u_1(z) = (A\,\mathrm{e}^{-\mathrm{i}\lambda_1\bar{z}} + B\,\mathrm{e}^{\mathrm{i}\lambda_1\bar{z}}) + \phi_{12}\,(C\,\mathrm{e}^{-\mathrm{i}\lambda_2\bar{z}} + D\,\mathrm{e}^{\mathrm{i}\lambda_2\bar{z}}), \qquad 0\leqslant\bar{z}\leqslant 1 \tag{4b}$$

$$u_2(z) = \phi_{21}\,(A\,\mathrm{e}^{-\mathrm{i}\lambda_1\bar{z}} + B\,\mathrm{e}^{\mathrm{i}\lambda_1\bar{z}}) + (C\,\mathrm{e}^{-\mathrm{i}\lambda_2\bar{z}} + D\,\mathrm{e}^{\mathrm{i}\lambda_2\bar{z}}), \qquad 0\leqslant\bar{z}\leqslant 1 \tag{5}$$

where

$$\lambda_1 = [a + (b^2 - \bar{\omega}^2\bar{c}_1\bar{c}_2)^{1/2}]^{1/2}, \qquad \lambda_2 = [a - (b^2 - \bar{\omega}^2\bar{c}_1\bar{c}_2)^{1/2}]^{1/2} \tag{6a, b}$$

in which

$$a = \tfrac{1}{2}[(\bar{\omega}\bar{\beta}_1/\beta_1)^2 + (\bar{\omega}\bar{\beta}_1/\beta_2)^2 - \mathrm{i}\,\bar{\omega}\,(\bar{c}_1+\bar{c}_2)] \tag{7a}$$

$$b = \tfrac{1}{2}[(\bar{\omega}\bar{\beta}_1/\beta_1)^2 - (\bar{\omega}\bar{\beta}_1/\beta_2)^2 - \mathrm{i}\,\bar{\omega}\,(\bar{c}_1-\bar{c}_2)] \tag{7b}$$

and

$$\left(\frac{\beta_1}{\bar{\beta}_1}\right)^2\bar{c}_1 = \left(\frac{\rho_2}{\rho_1}\right)\left(\frac{\beta_2}{\bar{\beta}_1}\right)^2\bar{c}_2 = \pi\,\xi_o \tag{8}$$

The factors ϕ_{12} and ϕ_{21} appearing in Equations (4b) and (5) are given by

$$\frac{\phi_{12}}{\bar{c}_1} = -\frac{\phi_{21}}{\bar{c}_2} = -\frac{\mathrm{i}\,\bar{\omega}}{b + (b^2 - \bar{\omega}^2\bar{c}_1\bar{c}_2)^{1/2}} \tag{9}$$

For $\omega\geqslant 0$ we select $Re(b^2 - \bar{\omega}^2\bar{c}_1\bar{c}_2)^{1/2}\geqslant 0$, $Re\,\lambda_1\geqslant 0$ and $Re\,\lambda_2\geqslant 0$.

The shear forces $S_j(z)$ along the beams can be calculated by

$$S_j(z) = (\rho_j\beta_j^2/l_2)u_j' \quad (j=1,2) \tag{10}$$

where again the prime denotes derivative with respect to $\bar{z}=z/l_2$.

The unknown constants A, B, C, D, E and F appearing in the expressions for $u_1(z)$ and $u_2(z)$ are determined by the boundary conditions at the bottom and top of both structures

$$u_1(0) = u_g, \qquad u_2(0) = u_g \tag{11a, b}$$

$$S_1(l_1) = 0, \qquad S_2(l_2) = 0 \tag{12a, b}$$

and by the continuity conditions

$$u_1(l_2^-) = u_1(l_2^+), \qquad S_1(l_2^-) = S_1(l_2^+) \tag{13a, b}$$

for the first structure at $z=l_2$. The resulting system of six equations in six unknowns can be solved analytically or numerically and the transfer functions $u_1(z)/u_g$, $u_2(z)/u_g$, $S_1(z)/(-\mathrm{i}\,\omega\rho_1\bar{\beta}_1u_g)$ and $S_2(z)/(-\mathrm{i}\,\omega\rho_1\bar{\beta}_1u_g)$ can

be calculated for a range of frequencies. These transfer functions depend on the dimensionless parameters $\bar{\omega} = \omega l_2/\bar{\beta}_1$, $\bar{\beta}_1/\bar{\beta}_2$, ρ_1/ρ_2, l_1/l_2, ξ_1, ξ_2 and ξ_o. Results in the time domain for a prescribed motion of the base u_g can then be obtained by Fourier synthesis.

In the particular case in which $\bar{\beta}_1 = \bar{\beta}_2$, $\xi_1 = \xi_2$ and $\rho_1 = \rho_2$, a simple solution for the response of both structures can be obtained.[9]

CHARACTERISTICS OF THE FREQUENCY RESPONSE

To illustrate the characteristics of the frequency response we consider first two uniform shear walls characterized by $l_1 = 80$ m, $l_2 = 40$ m, $\bar{\beta}_1 = \bar{\beta}_2 = 160$ m/sec, $\rho_1 = 2\rho_2$ and $\xi_1 = \xi_2 = 0{\cdot}02$. The structures represent a 20-storey building with a fundamental frequency $f_1 = \bar{\beta}_1/4l_1 = 0{\cdot}5$ Hz adjacent to a 10-storey building with a fundamental frequency $f_1 = \bar{\beta}_2/4l_2 = 1{\cdot}0$ Hz. The two structures are interconnected by distributed dampers with damping constant per unit length $c = \pi\rho_1\bar{\beta}_1\xi_o/l_2$ where ξ_o is a dimensionless interconnection damping constant. The total damping constant cl_2 is then given by $c\,l_2 = 2M_1\omega_1\xi_o$ where $M_1 = \rho_1 l_1$ and $\omega_1 = 2\pi f_1 = (\pi\bar{\beta}_1/2l_1)$ are the total mass and the fundamental frequency of the first structure.

Figure 2 shows the amplitudes of the transfer functions for the absolute displacements $|u_1(l_1)/u_g|$ and $|u_2(l_2)/u_g|$ at the top of the first and second structure and for the relative displacement $|[u_1(l_2) - u_2(l_2)]/u_g|$ at the top of the second structure. The results are presented for five values of ξ_o ranging from $\xi_o = 0{\cdot}0$ to $4{\cdot}0$. The results in Figure 2(a) for $|u_1(l_1)/u_g|$ indicate that the interconnecting damping has a strong effect on the

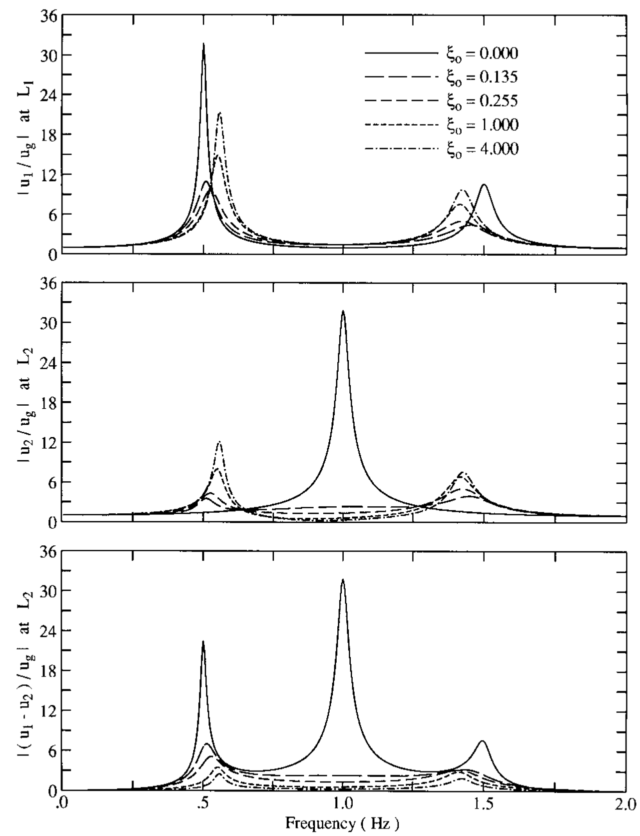

Figure 2. Transfer functions for the absolute displacements at the top of the first and second structures and for the relative displacement between the two structures for different interconnection dampers ($l_1 = 80$ m, $l_2 = 40$ m, $\bar{\beta}_1 = \bar{\beta}_2 = 160$ m/sec, $\rho_1 = 2\rho_2$, $\xi_1 = \xi_2 = 0{\cdot}02$)

peak amplitudes of the transfer functions in the vicinity of the first and second modes of the first structure (0·5 and 1·5 Hz, respectively). A second observation is that optimal values exist for ξ_o that minimize the peak amplitudes of the first and second modes. It appears from Figure 2(a) that values of $\xi_o = 0{\cdot}255$ and $\xi_o = 0{\cdot}135$ approximately minimize the peak response at the first and second modes, respectively. For $\xi_o = 0{\cdot}255$ the peak amplitudes in the first and second modes of the first structure are reduced by 70 and 53 per cent, respectively. For $\xi_o = 0{\cdot}135$ the corresponding reductions are 66 and 58 per cent, respectively. Increasing the interconnecting damping beyond the optimal values only increases the response on the first two modes.

The results for the transfer function $|u_2(l_2)/u_g|$ at the top of the second structure are shown in Figure 2(b). The response of the second structure with first and second modes at $f_1 = 1{\cdot}0$ and 3·0 Hz also shows the existence of an optimal damping. Significant reductions in the peak amplitudes are obtained for these optimal interconnecting damping values. Finally, Figure 2(c) shows the transfer function for the relative displacement between the two structures at the top of the second structure $[u_1(l_2) - u_2(l_2)]/u_g$. In this case, the relative displacement is reduced by increasing ξ_o.

OPTIMAL INTERCONNECTING DAMPING

The previous example shows that interconnection damping can significantly reduce the response of one or both of the structures at certain frequencies. The optimal value for the interconnecting damping ξ_o depends on the properties of the structures and on the particular mode being controlled.

The dimensionless transfer functions $u_1(l_1)/u_g$ is a function of the dimensionless parameters $\bar{\omega}$, l_1/l_2, ρ_1/ρ_2, $\bar{\beta}_1/\bar{\beta}_2$, ξ_1, ξ_2 and ξ_o. The peak value of the transfer function in the vicinity of the fundamental frequency is then only a function of l_1/l_2, ρ_1/ρ_2, $\bar{\beta}_1/\bar{\beta}_2$, ξ_1, ξ_2 and ξ_o. Consequently, the optimum value of ξ_o which minimizes the peak value of the transfer function $u_1(l_1)/u_g$ is only a function of l_1/l_2, ρ_1/ρ_2, $\bar{\beta}_1/\bar{\beta}_2$, ξ_1 and ξ_2. For most structures it can be assumed that $\bar{\beta}_1 \approx \bar{\beta}_2 \approx 160$ m/sec, and we also assume here that $\xi_1 = \xi_2$. The main parameters controlling the optimal values of the normalized interconnecting damping ξ_o are then the height ratio l_1/l_2 and the mass per unit height ratio ρ_1/ρ_2. In general, ρ_1/ρ_2 increases as a function of l_1/l_2 but there is considerable scatter as the slenderness of buildings varies over a wide range.

The optimal values ξ_{o1} and ξ_{o2} which minimize the peak amplitudes of the transfer function $|u_1(l_1)/u_g|$ at the top of the first structure in the vicinity of the first and second modes for the case $\bar{\beta}_1 = \bar{\beta}_2$, $\xi_1 = \xi_2 = 0{\cdot}02$ are listed in Tables I and II and are shown in Figures 3(a) and 3(d) as functions of the height ratio l_2/l_1 for three values of ρ_1/ρ_2 corresponding to $\rho_1/\rho_2 = 1{\cdot}0$, 2·0 and 4. When the two structures have similar heights the optimum values for ξ_o are small and can be easily accomplished. On the other hand, when the height difference is large ($l_2/l_1 < 0{\cdot}2$) much larger values for ξ_o are required to minimize the response of the taller structure. The optimum damping ratios for the first and second mode coincide at values of $\xi_o = 0{\cdot}12$, 0·19 and 0·28 for $l_2/l_1 = 0{\cdot}55$, 0·57 and 0·58 and $\rho_1/\rho_2 = 4{\cdot}0$, 2·0 and 1·0, respectively.

The peak amplitudes of $|u_1(l_1)/u_g|$ in the vicinity of the first mode of the taller structure for the optimum damping values ξ_{o1} and ξ_{o2} which minimize the response in the first and second modes are shown in Figures 3(b) and 3(e), respectively. For $\xi_o = \xi_{o1}$ (Figure 3(b)), large reductions of the response are obtained for most values of l_2/l_1 and, particularly, for values of $l_2/l_1 \approx 0{\cdot}5$. For structures with similar heights ($l_1/l_2 > 0{\cdot}95$) or very different heights ($l_2/l_1 < 0{\cdot}1$) only small response reductions are achieved. If the damping ξ_{o2} which minimizes the response in the second mode is used (Figure 3(e)), then the reductions of the peak response in the first mode are also significant with the exception of the cases $l_2/l_1 < 0{\cdot}1$, $l_2/l_1 \approx 0{\cdot}3$ and $l_2/l_1 > 0{\cdot}9$.

The peak amplitudes of $|u_1(l_1)/u_g|$ in the vicinity of the second mode of the taller structure for the optimal values ξ_{o1} and ξ_{o2} are shown in Figures 3(c) and 3(f), respectively. If the optimum damping values ξ_{o2} are used (Figure 3(f)) then strong reduction in response are obtained for all values of l_2/l_1 with the exception of cases $l_2/l_1 < 0{\cdot}1$, $l_2/l_1 \approx 0{\cdot}33$ and $l_2/l_1 > 0{\cdot}95$. The largest reduction occurs for $l_2/l_1 \approx 0{\cdot}70$. If the damping ξ_{o1} is used (Figure 3(c)) then reductions of the response in the second mode are obtained for $l_2/l_1 < 0{\cdot}25$ and $l_2/l_1 > 0{\cdot}4$. In the range $0{\cdot}25 < l_2/l_1 < 0{\cdot}4$, amplification of the second mode response by as much as 80 per cent can be obtained.

Table I. Optimal interconnecting damping for first mode and equivalent damping ratio for first mode for $\rho_1/\rho_2 = 4{\cdot}0$, $2{\cdot}0$ and $1{\cdot}0$ ($\bar{\beta}_1 = \bar{\beta}_2$, $\xi_1 = \xi_2 = 0{\cdot}02$)

l_2/l_1	ξ_{o1} (a)	ξ_{o1} (b)	ξ_{o1} (c)	$\hat{\xi}_1$ (per cent) (a)	$\hat{\xi}_1$ (per cent) (b)	$\hat{\xi}_1$ (per cent) (c)
0·00	—	—	—	2·00	2·00	2·00
0·10	1·043	1·711	2·682	2·82	3·40	4·13
0·20	0·493	0·804	1·273	3·62	4·79	6·36
0·30	0·310	0·525	0·788	4·30	6·05	8·55
0·40	0·215	0·358	0·541	4·80	7·04	10·44
0·50	0·151	0·255	0·374	5·04	7·59	11·69
0·58	0·111	0·183	0·279	5·02	7·60	12·05
0·60	0·103	0·175	0·255	4·98	7·56	12·00
0·70	0·072	0·111	0·167	4·61	6·89	11·21
0·80	0·040	0·064	0·095	3·93	5·64	9·040
0·90	0·016	0·032	0·040	3·02	3·96	5·56
1·00	0·000	0·000	0·000	2·00	2·00	2·00

Note: (a) $\rho_1/\rho_2 = 4{\cdot}0$, (b) $\rho_1/\rho_2 = 2{\cdot}0$, (c) $\rho_1/\rho_2 = 1{\cdot}0$.

Table II. Optimal interconnecting damping for second mode and equivalent damping ratio for first mode for $\rho_1/\rho_2 = 4{\cdot}0$, $2{\cdot}0$ and $1{\cdot}0$ ($\bar{\beta}_1 = \bar{\beta}_2$, $\xi_1 = \xi_2 = 0{\cdot}02$)

l_2/l_1	ξ_{o2} (a)	ξ_{o2} (b)	ξ_{o2} (c)	$\hat{\xi}_1$ (per cent) (a)	$\hat{\xi}_1$ (per cent) (b)	$\hat{\xi}_1$ (per cent) (c)
0·00	—	—	—	2·00	2·00	2·00
0·10	0·302	0·503	0·764	2·45	2·77	3·15
0·20	0·103	0·175	0·271	2·64	3·10	3·68
0·30	0·016	0·024	0·040	2·22	2·34	2·57
0·40	0·032	0·056	0·080	2·76	3·34	3·93
0·50	0·080	0·135	0·191	4·40	6·27	8·48
0·58	0·127	0·199	0·279	5·01	7·62	12·05
0·60	0·135	0·215	0·302	4·90	7·46	12·06
0·70	0·183	0·302	0·438	3·67	4·86	6·42
0·80	0·159	0·247	0·342	2·88	3·49	4·13
0·90	0·064	0·095	0·127	2·51	2·86	3·22
1·00	0·000	0·000	0·000	2·00	2·00	2·00

Note: (a) $\rho_1/\rho_2 = 4{\cdot}0$, (b) $\rho_1/\rho_2 = 2{\cdot}0$, (c) $\rho_1/\rho_2 = 1{\cdot}0$.

In addition to the optimal values ξ_{o1} and ξ_{o2}, Tables I and II list estimates of the apparent damping ratio $\hat{\xi}_1$ for the first mode achieved by introduction of the optimum interconnection dampers. These estimates were obtained from the width of the transfer functions $|u_1(l_1)/u_g|$ in the vicinity of the first mode. Apparent damping ratios for the first mode as high as 5, 8 and 12 per cent for $\rho_1/\rho_2 = 4{\cdot}0$, $2{\cdot}0$ and $1{\cdot}0$, respectively, can be obtained. In particular, for $l_2/l_1 = 0{\cdot}58$ and $\rho_1/\rho_2 = 1{\cdot}0$, the optimum value $\xi_o = 0{\cdot}279$ leads to an apparent damping ratio of 12·1 per cent for the first mode. This value is considerably higher than the assumed internal damping ratios of both structures ($\xi_1 = \xi_2 = 2$ per cent).

Since the optimal values for ξ_o depend on l_1/l_2 and ρ_1/ρ_2 the optimal value for the total damping constant $c\,l_2 = \pi\rho_1\bar{\beta}_1\xi_o$, which is also a measure of the total number of dampers required, is then mostly a function of ρ_1, l_1/l_2 and ρ_1/ρ_2. Considering a representative value of $\rho_1 \approx 2\times 10^5$ kg/m for a 50-storey building and $\bar{\beta}_1 \approx 160$ m/sec, the resulting value of cl_2 for $\xi_o = 0{\cdot}374$ is $c\,l_2 \approx 4\times 10^7$ kg/sec. This damping constant can

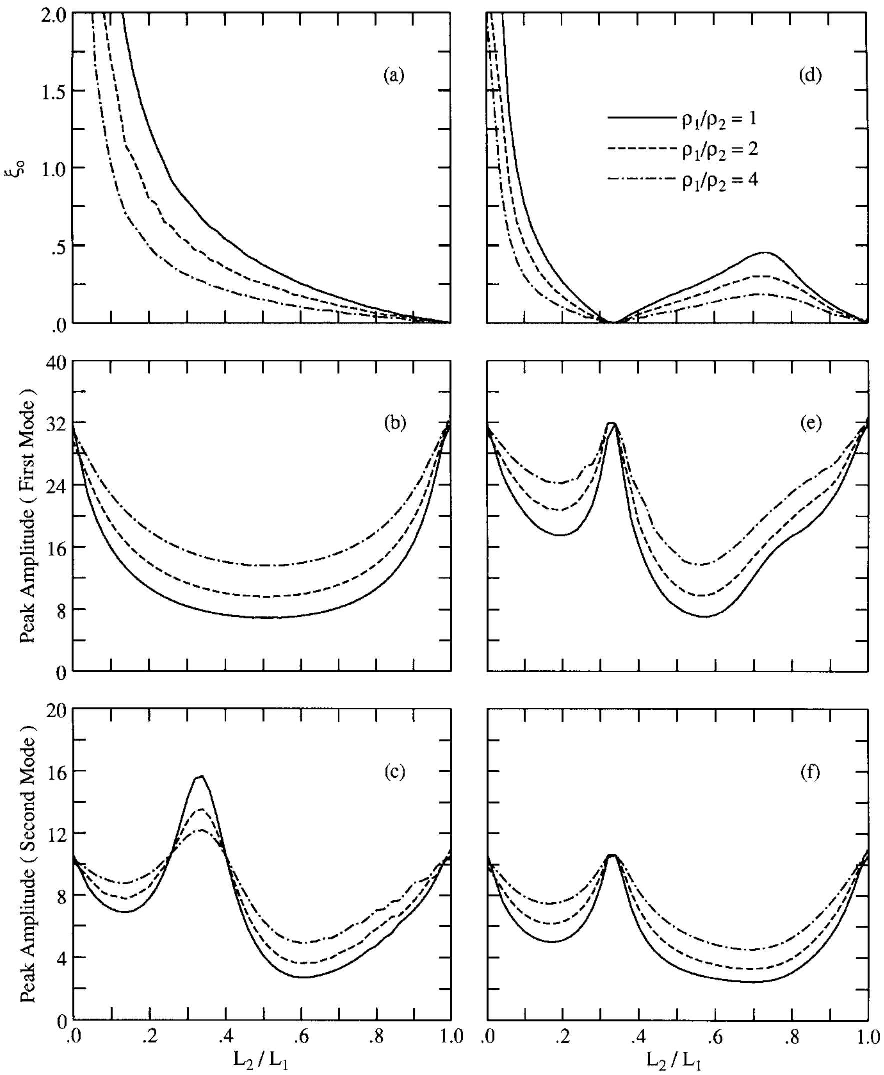

Figure 3. Optimal damping values ξ_{o1} and ξ_{o2} to minimize response in the first (a) and second (d) modes of the taller structure. Peak amplitudes of $|u_1(l_1)/u_g|$ in the vicinity of the first mode of the taller structure for ξ_{o1} (b) and ξ_{o2} (e). Peak amplitudes of $|u_1(l_1)/u_g|$ in the vicinity of the second mode of the taller structure for ξ_{o1} (c) and ξ_{o2} (f). [$\bar{\beta}_1 = \bar{\beta}_2$, $\xi_1 = \xi_2 = 0{\cdot}02$, $\rho_1/\rho_2 = 1{\cdot}0$, $2{\cdot}0$ and $4{\cdot}0$]

be achieved by the use of about 10 cylindrical dampers with diameter $D = 15$ cm, gap $d = 0{\cdot}1$ mm, length $l = 10$ cm and viscosity $\mu = 5 \times 10^{-3}$ kg/m sec.

Values for the optimal interconnecting damping ξ_o for structures with higher internal damping ($\xi_1 = \xi_2 = 0{\cdot}05$) are not very different from those obtained for $\xi_1 = \xi_2 = 0{\cdot}02$ but the relative reductions in response, although still significant, are not as large as those for more lightly damped structures. Numerical results can be found in a report by the authors.[9]

CHARACTERISTICS OF THE RESPONSE IN THE TIME DOMAIN

Next, we illustrate the effects of interconnecting dampers on the response of adjacent structures in the time domain. For this purpose we consider first the case of two structures characterized by $l_1 = 40$ m, $l_2 = 20$ m, $\bar{\beta}_1$

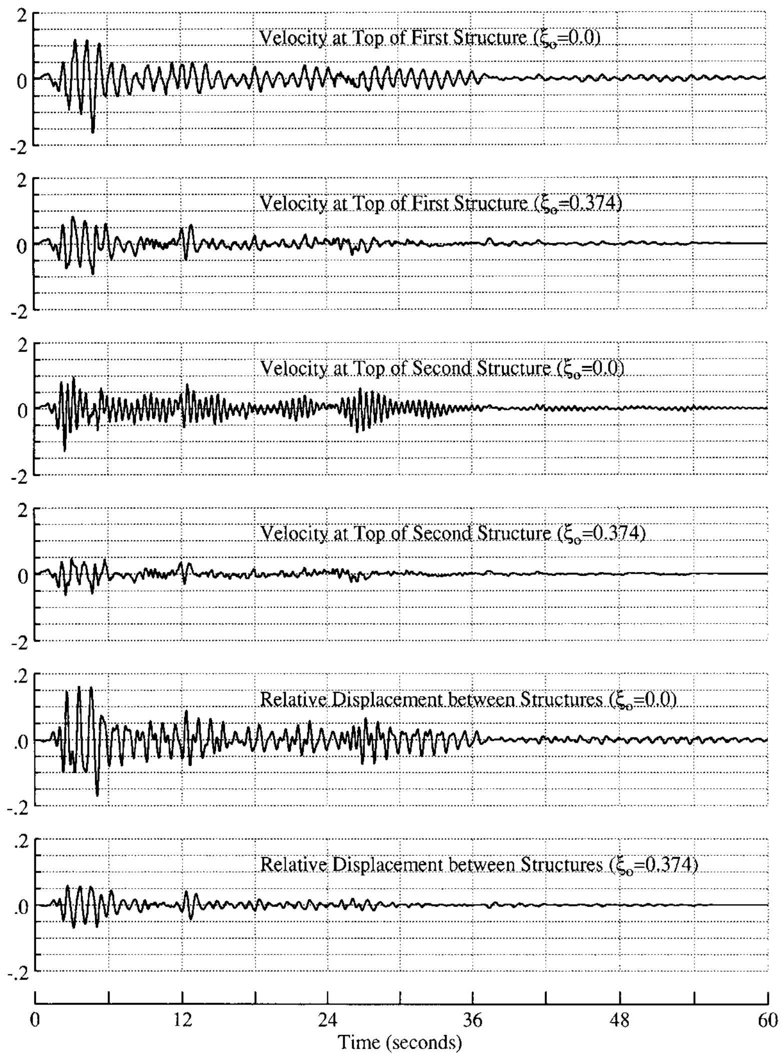

Figure 4. Velocity response (m/sec) at the top of the first and second structures and relative displacement (m) between the two structures for a 10-storey building adjacent to a five-storey building subjected to the NS El Centro 1940 ground motion ($l_1 = 40$ m, $l_2 = 20$ m, $\bar{\beta}_1 = \bar{\beta}_2 = 160$ m/sec, $\rho_1 = \rho_2$, $\xi_1 = \xi_2 = 0{\cdot}02$)

$= \bar{\beta}_2 = 160$ m/sec, $\rho_1 = \rho_2$ and $\xi_1 = \xi_2 = 0{\cdot}02$ when subjected to a filtered version of the *NS* El Centro 1940 excitation (peak acceleration $= 3{\cdot}416$ m/sec^2, peak velocity $= 0{\cdot}35$ m/sec). The two structures represent a 10-storey building adjacent to a 5-storey building. The fundamental frequencies of the structures are 1 and 2 Hz, respectively. In this example, both structures are assumed to have the same mass per floor. The same results would be obtained if the 10-storey building is flanked by two identical five-storey buildings each with half the mass per floor of the taller building. In this case, distributed dampers with constants $c/2$ would connect each five-storey building to the main building.

The time histories of the velocity responses $\dot{u}_1(l_1)$ and $\dot{u}_2(l_2)$ at the top of both structures and of the relative displacement $[u_1(l_2) - u_2(l_2)]$ between both structures at the top of the shorter building are shown in Figure 4. Results for the case of optimal interconnecting damping defined by $\xi_o = \xi_{o1} = 0{\cdot}374$ are compared with those

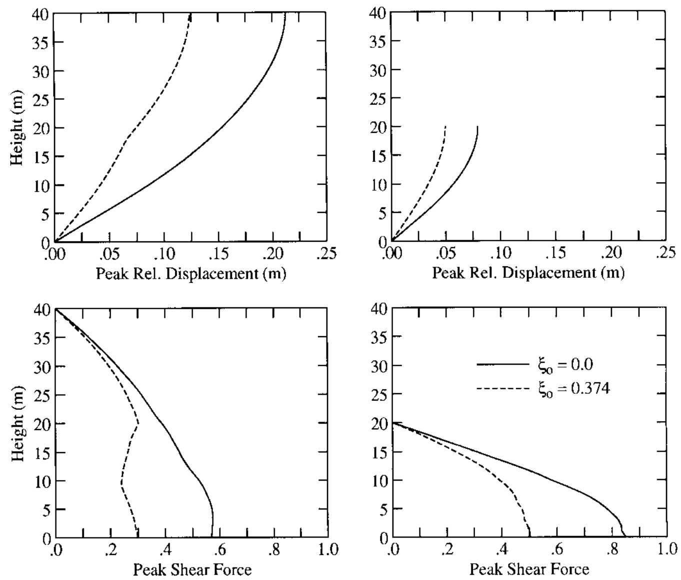

Figure 5. Distributions of peak displacements relative to the base (m) and peak shear forces (normalized by M_1 g and M_2 g) along the heights of both structures for El Centro 1940 excitation ($l_1 = 40$ m, $l_2 = 20$ m, $\bar{\beta}_1 = \bar{\beta}_2 = 160$ m/sec, $\rho_1 = \rho_2$, $\xi_1 = \xi_2 = 0{\cdot}02$)

for the case of uncoupled structures ($\xi_o = 0$). The results in Figure 4 indicate that the peak absolute velocities at the top of the structures were reduced by 43 and 50 per cent, respectively. The peak relative displacement between the two structures was reduced by 60 per cent.

The distributions of peak displacements relative to the base $[u_i(z) - u_g]$ and peak shear forces $S_1(z)/M_1$g and $S_2(z)/M_2$g normalized by the weights of both structures are presented in Figure 5 for the cases $\xi_o = 0$ and $\xi_o = 0{\cdot}374$. Clearly, the introduction of the optimum interconnecting dampers has reduced the peak values of the different response components throughout both structures.

Results for additional cases including pairs of tall structures and cases in which the taller structure has a higher mass per floor than the shorter structure are summarized in Table III. In the coupled case ($\xi_o \neq 0$) the dimensionless interconnecting damping constant ξ_o was selected to correspond to the optimal value ($\xi_o = 0{\cdot}374$ for $l_2/l_1 = 0{\cdot}5$ and $\rho_1/\rho_2 = 1{\cdot}0$ and $\xi_o = 0{\cdot}255$ for $l_2/l_1 = 0{\cdot}5$ and $\rho_1/\rho_2 = 2{\cdot}0$). For comparison, the results for uncoupled structures are also listed in Table III. For taller paired structures subjected to the same El Centro 1940 excitation and for heavier primary structures, the beneficial effects of interconnection dampers on the response of the taller structure may not be as pronounced as those for the case of a 10-storey building adjacent to a five-storey building. The results in Table III for a 20-storey building ($l_1 = 80$ m) adjacent to a 10-storey building ($l_2 = 40$ m) and for a 50-storey ($l_1 = 200$ m) building adjacent to a 25-storey ($l_2 = 100$ m) building indicate that slight increases in the peak accelerations or velocities at the top of the taller structure may be obtained. This is a result of the frequency content of the El Centro excitation being centred on some of the higher modes of the taller structure. Even in these cases, the base shear force for the taller structure, all of the response components for the second (shorter) structure and the relative displacement between the two structures are reduced significantly by the presence of interconnecting dampers. We note here that for seismic excitation with more pronounced low-frequency components, such as those for large magnitude earthquakes, the beneficial effects of interconnecting dampers would also extend to taller structures in the 50- to 100-storey class.

The large reductions by 50–60 per cent of the relative displacement between the two structures listed in Table III indicate that the stroke of the dampers would be reasonable and that this approach can also be used to mitigate the problem of pounding between adjacent buildings.

Table III. Effect of interconnection damping on the peak amplitudes of the response of 10, 20 and 50-storey structures adjacent to 5, 10 and 25-storey structures for El Centro 1940 excitation ($\bar{\beta}_1 = \bar{\beta}_2 = 160$ m/sec, $\xi_1 = \xi_2 = 0{\cdot}02$)

ξ_o	$\frac{\rho_1}{\rho_2}$	$\ddot{u}_1(l_1)$ (m/sec^2)	$\dot{u}_1(l_1)$ (m/sec)	$\dot{u}_2(l_2)$ (m/sec)	$\frac{S_1(0)}{M_1 g}$	$\frac{S_2(0)}{M_2 g}$	$u_1(l_2) - u_2(l_2)$ (m)
			(a) $l_1 = 40$ m, $l_2 = 20$ m				
0·000	—	9·885	1·631	1·298	0·570	0·849	0·171
0·255	2·0	9·294	1·192	0·714	0·387	0·612	0·077
0·374	1·0	8·719	0·931	0·644	0·299	0·502	0·068
			(b) $l_1 = 80$ m, $l_2 = 40$ m				
0·000	—	5·772	1·117	1·631	0·224	0·570	0·283
0·255	2·0	5·277	1·071	0·745	0·172	0·268	0·108
0·374	1·0	5·871	1·029	0·710	0·157	0·239	0·111
			(c) $l_1 = 200$ m, $l_2 = 100$ m				
0·000	—	4·678	0·590	1·114	0·046	0·177	0·448
0·255	2·0	4·598	0·603	0·590	0·033	0·071	0·152
0·374	1·0	4·566	0·636	0·602	0·031	0·074	0·180

CONCLUSIONS

The optimal values for the distribution of passive dampers interconnecting two adjacent structures of different heights have been determined. The optimal damping values minimize the peak amplitudes of the transfer functions for the reponse at the top of the taller structure in the vicinity of the first and second modes of the structure.

If $0{\cdot}4 < l_2/l_1 < 1{\cdot}0$, where l_2 is the height of the shorter structure, the use of the interconnecting dampers that minimize the response in the first or second mode leads to significant reductions in the peak amplitudes of both modes. In the case $l_2/l_1 = 0{\cdot}5$ and $\rho_1/\rho_2 = 1{\cdot}0$ the optimal damping for the first mode leads to a 78 per cent response reduction in that mode and to a 62 per cent reduction in the response of the second mode. If $0{\cdot}25 < l_2/l_1 < 0{\cdot}4$, the optimal damper which minimizes the first mode response may lead to an increase in the response in the second mode by as much as 48 per cent. The dampers which minimize the second mode response lead to response reductions in both the first and second modes. Finally, for $l_2/l_1 < 0{\cdot}25$, response reductions for both modes are obtained at the expense of large values for the interconnecting damping constants.

The optimal value for the dimensionless damping constant ξ_o depends on the relative height l_2/l_1, the relative mass per unit height ρ_1/ρ_2 and on the damping ratios ξ_1 and ξ_2. The required total number of interconnecting dampers $cl_2 = \pi\rho_1\bar{\beta}_1\xi_o$ depends, in addition, on ρ_1 and $\bar{\beta}_1$.

In the case of adjacent structures with relative heights l_2/l_1 in the range from 0·50 to 0·80, and mass ratios $1 \leqslant \rho_1/\rho_2 \leqslant 2$ apparent damping ratios of 7·0 to 12 per cent for the first mode can be achieved by use of the optimal interconnecting dampers for the first mode. If the optimal interconnecting damping for the second mode is used, apparent damping ratios from 3·5 to 8·5 per cent for the first mode can be obtained. Larger effects can be obtained if the mass per floor of the shorter structure can be larger than that for the taller building.

The use of interconnecting dampers reduces drastically the relative displacement between the two structures without increasing the shear forces and, consequently, this approach can be applied to mitigate the problem of pounding between adjacent structures.

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